Transport in one-dimensional random resistor-superconductor mixtures with random distribution of resistor strength

1986 J. Phys. A: Math. Gen. 19 L419

(http://iopscience.iop.org/0305-4470/19/8/002)

J. Phys. A: Math. Gen. 19 (1986) L419-L423. Printed in Great Britain

LETTER TO THE EDITOR

Transport in one-dimensional random resistor–superconductor mixtures with random distribution of resistor strength

S Havlin†‡, A Bunde‡§ and J Kiefer||

† Department of Physics, Bar-Ilan University, Ramat-Gan, Israel
‡ Fakultät für Physik, Universität Konstanz, 7750 Konstanz, West Germany
§ Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA
|| PSL, DCRT, NIH, Bethesda, MD 20205, USA

Received 10 February 1986

Abstract. We study the problem of transport in linear random resistor-superconductor mixtures with a random distribution of resistor strength. The superconductors with a concentration p are represented in our model as short circuits. The resistor concentration is $(1-p)$ and their conductivity distribution is $p(\sigma)\sim\sigma^{-\alpha}$, $\alpha<1$. We find that for $\alpha>0$ the specific conductivity scales with the linear size L of the system as $L^{-\alpha/(1-\alpha)}(1-p)^{-1/(1-\alpha)}$. The mean square displacement $\langle x^2\rangle$ of a random walker in this system scales as $\langle x^2\rangle^{(2-\alpha)/(1-\alpha)2}\sim(1-p)^{-(2-\alpha)/(1-\alpha)}t$. In the presence of a bias field we obtain $\langle x\rangle^{1/(1-\alpha)}\sim(1-p)^{-1/(1-\alpha)}t$. We present an exact enumeration method to study diffusion on those systems; our numerical results confirm the above scaling relations.

The problem of transport on random resistor-superconducting networks (de Gennes 1980) has recently been studied extensively (Coniglio and Stanley 1984, Adler *et al* 1985, Bunde *et al* 1985, Sahimi and Saddiqui 1985, Leyvraz *et al* 1986). Several different models were presented by the above authors, including the short-circuit model for describing the transport properties of a random superconducting-resistor network. In the short-circuit model Monte Carlo results were not adequate to determine the exponents characterising the transport properties even in one dimension (Adler *et al* 1985). Very recently the problem of transport on a random distribution of conductors in a random resistor-isolator mixture was studied by Halperin *et al* (1985) and non-universal exponents were predicted which depend on the specific form of the conductor distribution.

In this letter we study in one dimension the problem of a random resistor-superconductor mixture with a random distribution of resistor strength. We study this problem analytically by using scaling arguments similar to those presented by Halperin *et al* (1985) and by simulations using an exact enumeration method. The concentration of the superconductors is assumed to be p and each one is represented by a short circuit between the two resistors on both sides of the superconductor. The concentration of resistors is $1-p$ and their conductances have the following distribution:

$$P_0(\sigma)\sim\sigma^{-\alpha}\qquad \alpha<1, 0\leqslant\sigma\leqslant 1. \tag{1}$$

An example of a physical system for which $\alpha\neq 0$ is as follows. Assume that all resistors are cylinders with the same length but with a constant distribution of diameters $0\leqslant a\leqslant 1$. Since $\sigma\sim a^2$, it follows that $p(\sigma)\sim \mathrm{d}a/\mathrm{d}\sigma\sim 1/\sqrt{\sigma}$, that is $\alpha=\frac{1}{2}$.

0305-4470/86/080419+05$02.50

The average conductivity Σ of a system of size L (L is the total number of superconductors and resistors) can be calculated through

$$\Sigma^{-1} = \sum_{i=1}^{l} \frac{1}{\sigma_i} \tag{2}$$

where l is the number of resistors, $l = L(1-p)$. Note that the superconducting parts have (by definition) zero resistivity and therefore do not contribute to the total resistivity $1/\Sigma$ in (2). Using (1) we obtain for $\alpha > 0$

$$\Sigma^{-1} = l \int_{\sigma_{\min}}^{1} \frac{1}{\sigma} P_0(\sigma)\, \mathrm{d}\sigma \sim l \int_{\sigma_{\min}}^{1} \sigma^{-\alpha-1}\, \mathrm{d}\sigma \sim l\sigma_{\min}^{-\alpha}. \tag{3}$$

Since $\sigma_{\min} \sim l^{-1/(1-\alpha)}$ (see e.g. Sen *et al* 1985) we obtain

$$\Sigma^{-1} \sim l^{1/(1-\alpha)} \tag{4}$$

and the specific conductivity becomes

$$G \equiv \Sigma L \sim L^{-\alpha/(1-\alpha)}(1-p)^{-1/(1-\alpha)}. \tag{5}$$

In the thermodynamic limit, $L \to \infty$ and hence $G \to 0$ for any $\alpha > 0$. For constant $P_0(\sigma)$, i.e. $\alpha = 0$, G scales as $\ln L$. For $\alpha < 0$, the integrals in (3) do not depend on l and we recover the conventional result $\Sigma^{-1} \sim L(1-p)$. Hence the specific conductivity is independent of L and α. The diffusion constant $D \equiv \langle x^2 \rangle / t \sim L^2/t$ can be calculated through the Einstein relation $D \sim G/(1-p)$ which yields

$$D \sim \begin{cases} L^{-\alpha/(1-\alpha)}(1-p)^{-(2-\alpha)/(1-\alpha)} & \alpha > 0 \\ (1-p)^{-2} & \alpha < 0. \end{cases} \tag{6}$$

Therefore the mean square displacement $\langle x^2 \rangle$ scales as

$$\langle x^2 \rangle^{(2-\alpha)/2(1-\alpha)} \sim (1-p)^{-(2-\alpha)/(1-\alpha)} t \qquad \alpha > 0 \tag{7a}$$

$$\langle x^2 \rangle \sim (1-p)^{-2} t \qquad \alpha < 0. \tag{7b}$$

Equation (7*a*) predicts, therefore, anomalous diffusion in one dimension with a diffusion exponent

$$d_{\mathrm{w}} = (2-\alpha)/(1-\alpha) \qquad \alpha \geqslant 0. \tag{8}$$

The special case $\alpha = -\infty$ (all resistors have the same conductivity $\sigma = 1$) coincides with a recent theoretical result of Leyvraz *et al* (1986). The special case $p = 0$ reduces to the known result of Alexander *et al* (1981) for diffusion in a one-dimensional system with a distribution of transition rates.

We have also studied the effect of a bias field on the diffusion. The time t for a random walker to travel along l resistor sites under the influence of the bias field is given by (see also Bunde *et al* 1986)

$$t \sim \sum_{i=1}^{l} \frac{1}{\sigma_i}. \tag{9}$$

Using similar scaling arguments as in (2)-(4) we obtain for the mean displacement $\langle x \rangle$ of the random walker

$$\langle x \rangle^{1/(1-\alpha)} \sim (1-p)^{-1/(1-\alpha)} t \qquad \alpha > 0. \tag{10}$$

In the following we describe a numerical method for simulation of diffusion on the short-circuit one-dimensional model for random resistor-superconductor mixtures. First we have generated a random linear lattice of L sites by choosing superconducting sites with probability p and resistor sites with probability $1-p$. The resistors are labelled progressively by $i = 1, 2, \ldots, l$ and their coordinates in the chain are $x(i)$. Every resistor i carries a number $m(i)$ which counts the successive superconducting sites located between resistors i and $i+1$. The actual difference in coordinates x_{jk} between two arbitrary resistors j and k is obtained from

$$x_{jk} \equiv x(j) - x(k) = \operatorname{sgn}(j-k) \sum_{i=j}^{k} [m(i)+1]. \tag{11}$$

We have chosen the transition rates $W_{i,i+1}$ between neighbouring resistors i and $i+1$ according to

$$W_{i+1,i} = W_{i,i+1} = \tfrac{1}{2}\delta_i^{1/(1-\alpha)} \tag{12}$$

where δ_i is a random number between zero and one. By (12), the rate distribution $P(W) \sim W^{-\alpha}$ is generated, corresponding to (1).

For investigating the diffusion properties of the resistor-superconductor mixture we have applied the exact enumeration method (Ben-Avraham and Havlin 1982, Majid *et al* 1984), which allows us to calculate exactly the distribution function $P(x, t)$ of a random walker for a given lattice and a fixed starting point $x_0 = x(i_0)$. Here, the superconductors act as short circuits and therefore the walker can step only on resistor sites (Adler *et al* 1985). We study the distribution function $P(x(i) - x(i_0), t)$. At $t = 0$, $P(x, 0) = \delta_{x,0}$ by definition. At $t = 1$, the walker steps with probability W_{i_0,i_0+1} to the resistor site i_0+1 located at $x(i_0+1)$ and with probability W_{i_0,i_0-1} to the resistor site i_0-1 located at $x(i_0-1)$. Hence

$$P(x,1) = \begin{cases} W_{i_0,i_0+1} & \text{for } x = x(i_0+1) - x_0 \\ 1 - W_{i_0,i_0+1} - W_{i_0,i_0-1} & \text{for } x = 0 \\ W_{i_0 i_0,-1} & \text{for } x = x(i_0-1) - x_0. \end{cases} \tag{13}$$

For $x \neq 0$ and $x \neq x(i_0 \pm 1) - x_0$, $P(x, 1) \equiv 0$. By iterating this procedure we find $P(x, 2)$, etc. From $P(x, t)$ we obtain the mean square displacement

$$\langle x^2(t)\rangle = \sum_i [x(i) - x(i_0)]^2 P[x(i) - x(i_0), t] \tag{14}$$

of the random walker with starting point at $x(i_0)$ for the considered configuration of resistors and superconductors.

In order to obtain the corresponding configurational averaged quantities one has to average over many lattice configurations. For our actual computations averages over (typically) 500 lattice configurations have been performed and up to 3×10^4 time steps have been considered.

First we have studied the case when all resistors are identical, i.e. $\alpha = -\infty$. Figure 1 shows our result for $\langle x^2(t)\rangle(1-p)^2$ as a function of t for several values of p. The observed data collapse clearly confirms (7*b*). Note that $\langle x^2\rangle$ is *linear* in t for small times $t = 1, 2, 3, \ldots,$ considered here. This normal random walk behaviour in $d = 1$ is in marked contrast to the results of Hong *et al* (1986) for higher-dimensional systems below the critical point. In $d = 2$, for random superconducting networks, there exist two different time regimes separated by a crossover time t_x. For $t \ll t_x$, $\langle r^2(t)\rangle = R_s^2$ while for $t \gg t_x$, $\langle r^2(t)\rangle \sim t$; R_s is the mean cluster radius. The crossover time is related

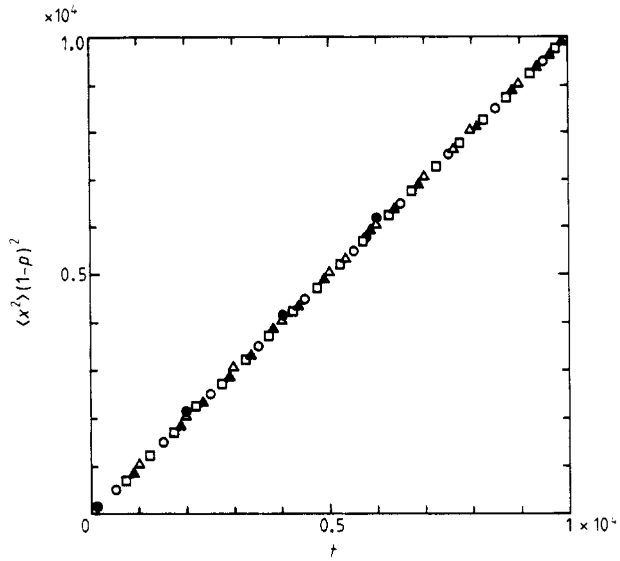

Figure 1. Plot of $\langle x^2\rangle(1-p)^2$ against t for resistors with conductivity 1 ($\alpha = -\infty$) for various concentrations p for superconductors: $p = 0.9$ ($\triangle$), $p = 0.95$ ($\square$), $p = 0.99$ ($\bigcirc$), $p = 0.996$ ($\blacktriangle$), $p = 0.999$ ($\bullet$). In the calculations the number of resistors was 10^4, corresponding to a lattice size of 10^7 for $p = 0.999$, and periodic boundary conditions have been employed.

to the fractal nature of the unscreened perimeter sites of superconducting clusters. Since in $d = 1$ all perimeter sites are unscreened, one has $\langle x^2(t)\rangle \sim t$ for all times considered.

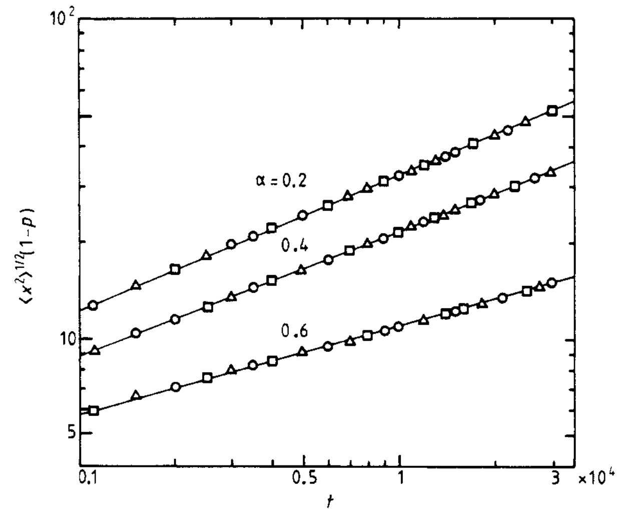

Figure 2. Plot of $\langle x^2\rangle^{1/2}(1-p)$ against t for different values of the distribution exponent α and various values of p: $p = 0.7$ ($\square$), $p = 0.8$ ($\triangle$), $p = 0.9$ ($\bigcirc$). The number of resistors was 500 and periodic boundary conditions have been used. From the slopes of the curves in the large time regime $10^4 \leqslant t \leqslant 3\times10^4$ we obtain d_w.

In figure 2 we have plotted, on a double logarithmic scale, $\langle x^2\rangle^{1/2}(1-p)$ as a function of t for several values of α and p. For $\alpha=0.2$, 0.4 and 0.6 we find $d_w=2.30\pm0.05$, 2.70 ± 0.05 and 3.50 ± 0.05 respectively. These values are in excellent agreement with (7a) and (8).

We thank D Stauffer for valuable discussions. S H gratefully acknowledges financial support from the USA-Israel Bi-National Foundation and from a Minerva Fellowship; A B gratefully acknowledges financial support from Deutsche Forschungsgemeinschaft.

References

Adler J, Aharony A and Stauffer D 1985 *J. Phys. A: Math. Gen.* **18** L130
Alexander S, Bernasconi I, Orbach R and Schneider W R 1981 *Rev. Mod. Phys.* **53** 175
Ben-Avraham D and Havlin S 1982 *J. Phys. A: Math. Gen.* **15** L691
Bunde A, Coniglio A, Hong D C and Stanley H E 1985 *J. Phys. A: Math. Gen.* **18** L137
Bunde A, Havlin S, Stanley H E, Trus B and Weiss G H 1986 *Preprint*
Coniglio A and Stanley H E 1984 *Phys. Rev. Lett.* **52** 1068
de Gennes P G 1980 *J. Physique Colloq.* **41** C3
Halperin B I, Feng S and Sen P N 1985 *Phys. Rev. Lett.* **54** 2391
Hong D, Stanley H E, Coniglio A and Bunde A 1986 *Phys. Rev.* B **33** in press
Leyvraz F, Adler J, Aharony A, Bunde A, Coniglio A, Stanley H E and Stauffer D 1986 *Preprint*
Majid I, Ben-Avraham D, Havlin S and Stanley H E 1984 *Phys. Rev.* B **30** 1626
Sahimi M and Saddiqui 1985 *J. Phys. A: Math. Gen.* **18** L727
Sen P N, Roberts J N and Halperin B I 1985 *Phys. Rev.* B **32** 3306